Design and implementation of non-overshooting MPC with terminal cost/terminal constraints for vehicle lateral stability control

Monish Dev Sudhakhar, M.S - Mechanical Engineering

Mentor: Dr. Yan Chen, Assistant Professor – Polytechnic Sch EGR Prgrms

Arizona State University

MOTIVATION

An effective method to guarantee vehicle safety is to design a vehicle driving safety algorithm that limits its states within a predefined stability region without passing the boundaries. A non-overshooting model predictive control (MPC) was preliminarily proposed to achieve that [1]. A new non-overshooting design is proposed in this work by considering MPC with terminal cost and terminal constraint ensuring stability and recursive feasibility [2]. The system output responses are studied using numerical examples for both linear and non-linear systems. It is finally applied to vehicle lateral dynamics to guarantee vehicle lateral stability.

METHODOLOGY

The non-overshooting control design can be implemented by considering the boundaries as references. In the next section, through the advantages of handling constraints, model predictive control (MPC) is utilized as an appropriate approach to develop a uniformed non-overshooting control design for general dynamic systems. Specifically, during the entire prediction horizon, constraints are applied at each sampling time to avoid the overshooting of system outputs. [1]

TERMINAL COST & TERMINAL CONSTRAINT

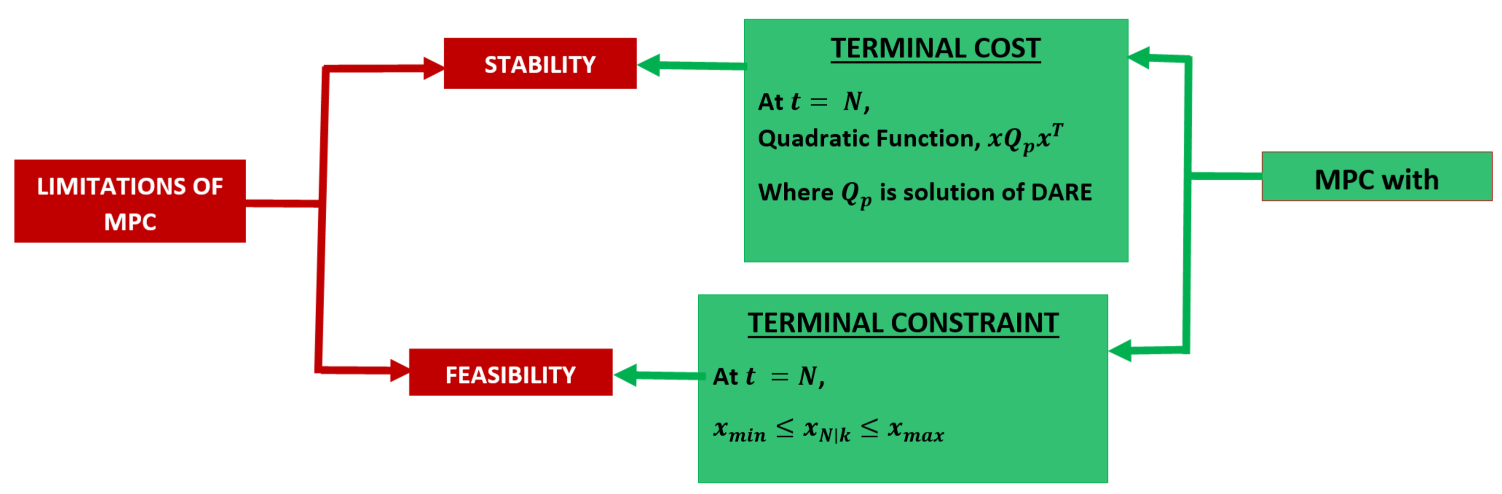

Fig 1 Need and Design - terminal cost /terminal constraint

LINEAR SYSTEM – SPRING MASS DAMPER

Consider a general linear system which is the form,

$\dot{x} = Ax + Bu$
$y = Cx + Du$ with,

$$A = \begin{bmatrix} 0 & 1 \\ -k/m & -b/m \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1/m \end{bmatrix}$$

$$C = [1 \quad 0], D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Where, $k = 0.1\ N/m$, $b = 0.1 N.s/m$, $m = 5\ kg$

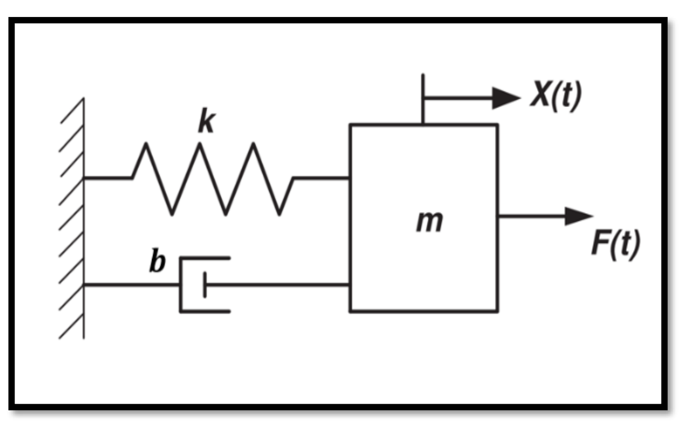

Fig 2 Spring Mass Damper

Here the optimization problem is,

$$J(k) = \sum_{j=1}^{N-1} \left[x(k+j\ |k) - x_{ref})^T]\right)] * Q * \left[x(k+j\mid k) - x_{ref}\right] + \sum_{j=1}^{P-1} \Delta u(k+j \mid k)^T * R * \Delta u(k+j\mid k)$$

Where $Q = diag[10 \quad 1]$, $R = 1$, Prediction Horizon, N= P = 10.
Sampling time Ts= 0.1s
Non-overshooting constraints are [1],
C1: $\boldsymbol{y_i(k+1|k) \le y_{i-ref}}$
C2: $\boldsymbol{y_i(k+j|k) \le y_{i-ref}}$, where $\boldsymbol{1 \le j \le N}$
C3: $\boldsymbol{y_i(k+N|k) \le y_{i-ref}}$ & $\boldsymbol{y_i(k+j|k) \le y_i(k+N|k)}$, where $\boldsymbol{1 \le j \le N-1}$
C4 : $\boldsymbol{y_i(k+N|k) \le y_{i-ref}}$ & $\boldsymbol{y_i(k+j|k) \le y_i(k+j+1|k)}$, where $\boldsymbol{1 \le j \le N-1}$
The System is simulated with the proposed constraints,

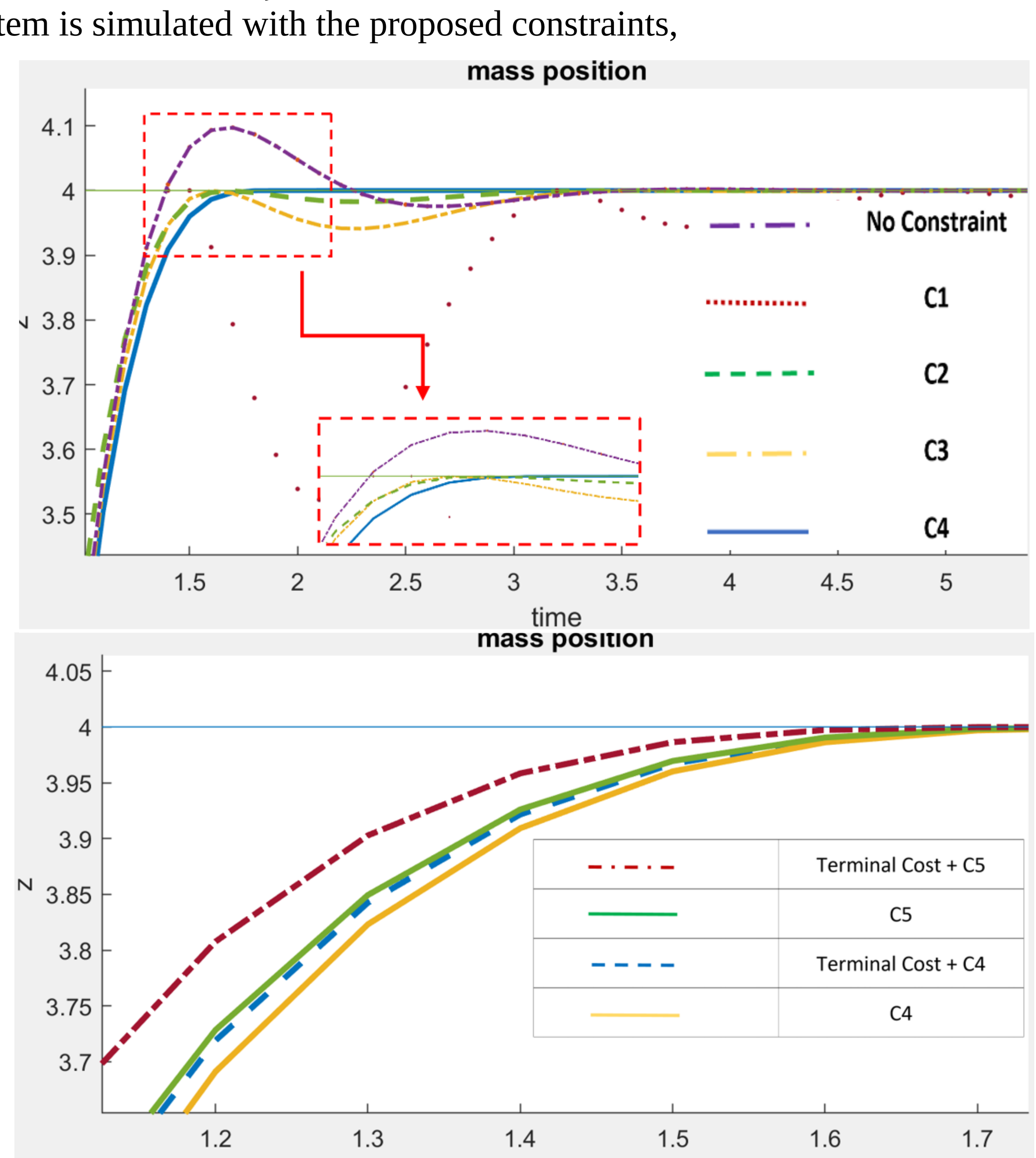

Fig 3 System response for linear MPC with non overshooting constraints, terminal cost & C5

We need to use the theory of terminal cost and terminal constraint to the proposed non overshooting design in order to ensure the mentioned characteristics [2]. The new overshooting design C5 is proposed using the terminal equality constraint as follows,

C5 : $\boldsymbol{y_i(k+N|k) - y_{i-ref} = 0}$ **&** $\boldsymbol{y_i(k+j|k) \le y_i(k+j+1|k)}$**, where** $\boldsymbol{1 \le j \le N-1}$

The addition of terminal cost is done in order to ensure the stability as it is the CLF.
The terminal cost is added at the last step of the horizon N, [2]

Terminal cost, $\boldsymbol{P(x(k+N|k)) = \left[x(k+N\ |k) - x_{ref})^T\right] * Q_p * \left[x(k+N\mid k) - x_{ref}\right]}$,
$\boldsymbol{Q_p = dare(A, B, Q, R)}$, ***Terminal constraint,*** $\boldsymbol{x(k+N\ |k) - x_{ref} = 0}$

NON-LINEAR SYSTEM – INVERTED CART PENDULUM

Non-linearity poses much complexity and difficulties with the choice the design parameters making it hard to tune the system. Consider a cart-pendulum system with the following dynamics,
$x = [z \quad \dot{z} \quad \theta \quad \dot{\theta}]^T$

$$\dot{x} = \begin{bmatrix} \dot{z} \\ \dfrac{F - K_d\dot{z} - m_pL\dot{\theta}^2\sin\theta + m_pg\sin\theta\cos\theta}{m_c + m_p\sin^2\theta} \\ \dot{\theta} \\ \dfrac{(F - K_d\dot{z} - m_pL\dot{\theta}^2\sin\theta)\cos\theta + (m_c + m_p)g\sin\theta}{L(m_c + m_p) - m_pL\cos^2\theta} \end{bmatrix}$$

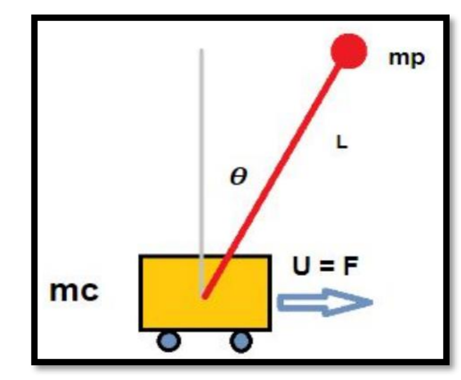

Fig 4 Inverted Cart Pendulum

where z, θ, and u are the cart position, pendulum angle, and input force applied on the cart. The parameter values are $m_p = m_c = 1$ kg, $L = 0.5$ m, and $K_d = 10$ N · s/m. The initial condition is $x(0) = [0 \quad 0 \quad -\pi \quad 0]^T$ and reference, $x_{\text{ref}} = [4 \quad 0 \quad 0 \quad 0]^T$. Weighting matrices are $Q = \begin{bmatrix} 10 & 0 \\ 0 & I_3 \end{bmatrix}$ and $R = [0.1]$. The system is simulated with the proposed constraints like the spring mass damper with N=P =10 & Ts= 0.1s. The only difference here is that the terminal penalty matrix cannot be solved by the solution of Riccati equation and hence it is assumed as a positive definite matrix, $Q_p = diag(50{,}5{,}10{,}1)$

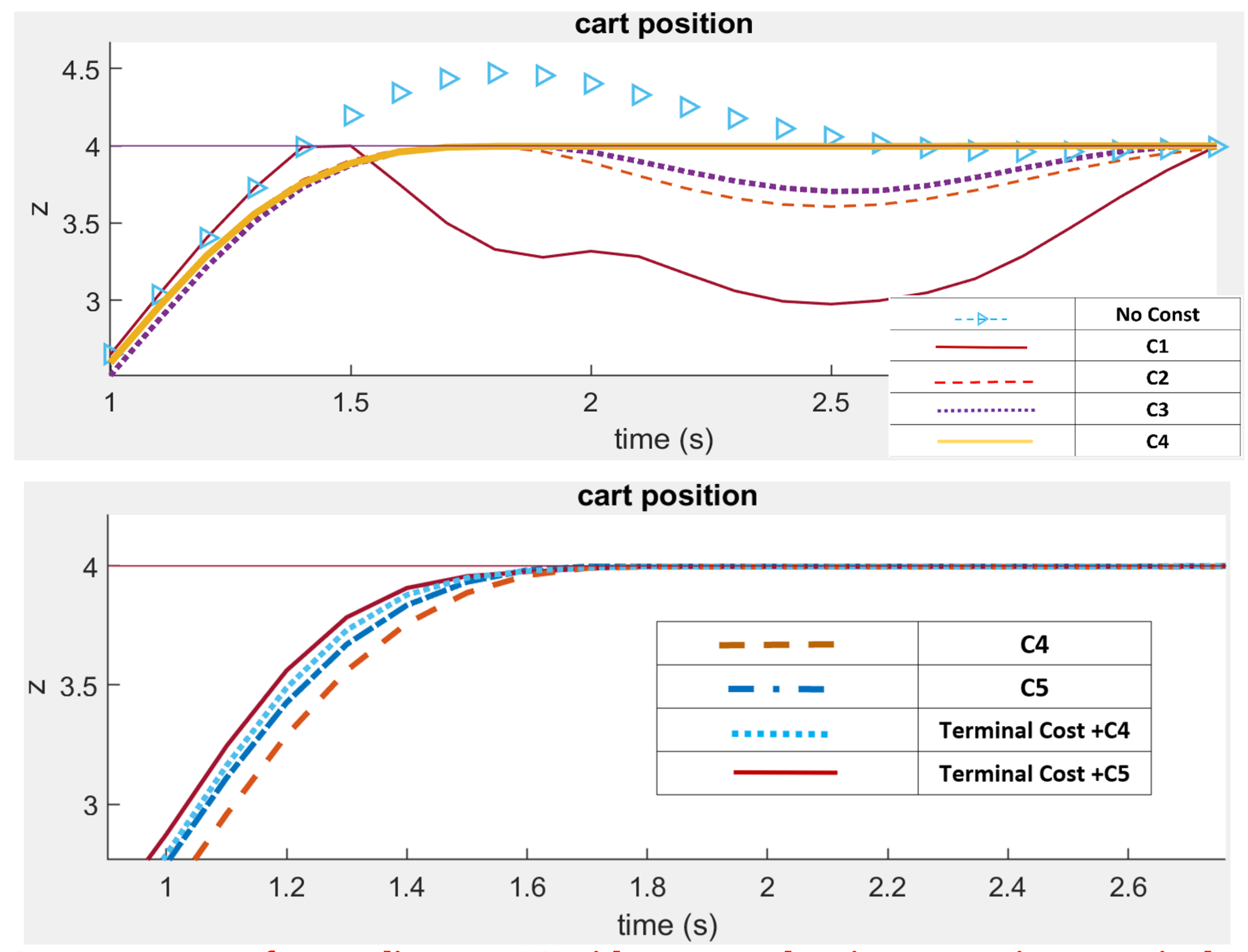

Fig 5 System response for non-linear MPC with non overshooting constraints. terminal cost & C5

From the simulation above we could infer the following characteristics,

Convergence: $\boldsymbol{Terminal\ cost + C5 > C5 > C4}$**> C3 > C2 > C1**

Settling Time: C1 > C2 > C3 > $\boldsymbol{C4 > C5 > Terminal\ cost + C5}$

VEHICLE STABILITY REGION

The vehicle stability region is constructed by considering the non-linear lateral dynamics as follows, [1]

$$m_v(\dot{V}_y + V_xr) = (F_{yfl} + F_{yffr})\cos\delta_f + F_{yrl} + F_{yrr} + F_{yAFS},$$
$$I_z\dot{r} = l_f[(F_{yfl} + F_{yfr})\cos\delta_f + F_{yAFS}] - l_r(F_{yyl} + F_{yrr}) + l_s(F_{yfl} - F_{yfr})\sin\delta_f,$$

where m_v, I_z, δ_f, V_x, V_y, and r are the vehicle mass, yaw moment of inertia, front steering angle, vehicle longitudinal velocity, lateral velocity, and yaw rate. l_s, l_f, and l_r are the wheel track, front wheelbase, and rear wheelbase, respectively. F_{yi} ($i = fl, fr, rl, rr$) are the lateral forces, which are calculated by 2D LuGre tire model, on four wheels, respectively. F_{yAFS} is the additional tire lateral force generated by the AFS control. [1]

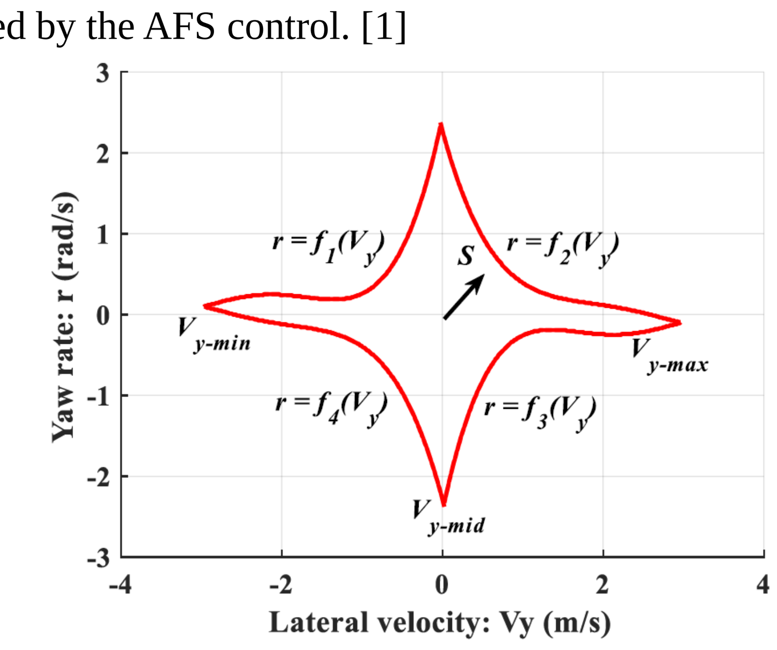

Fig 6 Vehicle Stability region

FUTURE WORK

The proposed non-overshooting design shall be applied considering the lateral dynamics subjected to shift in the stability region presented due to the varying steering input. It shall be verified with the actual vehicle parameters through MATLAB/Simulink and CarSim®.

ACKNOWLEDGEMENT

I would like to thank Dr. Yan Chen for his support and guidance. The understanding of proposed concepts were from EGR 560 - Vehicle Dynamics and Control coursework. Most of this research is based on,

1. Yiwen Huang & Yan Chen, "**Stability Region-based Vehicle Lateral Control Using Non-Overshooting MPC**" 2019 American Control Conference (ACC), Philadelphia, PA, USA, July 10-12, 2019.
2. F. Borrelli, A. Bemporad, M. Morari, **Predictive Control for linear and hybrid systems**, Date Oct 26, 2016
3. James B. Rawlings , David Q. Mayne, Moritz M. Diehl, **Model Predictive Control: Theory, Computation, and Design, 2nd Edition**